Networks of Evolutionary Processors with Subregular Filters

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Abstract

In this paper, we study networks of evolutionary processors where the filters are chosen as special regular sets. We show that the use of ordered, non-counting, power-separating, circular, and suffix-closed regular languages or languages that are combinational or reverse combinational is as powerful as the use of arbitrary regular languages. On the other hand, the use of only finite languages allows only the generation of regular languages, but not all regular languages can be generated. If only filters are used that are target sets of monoids then non-context-free languages can be generated but not all regular languages.

1. Introduction

A basic architecture for parallel and distributed symbolic processing, related to the Connection Machine ([11]) as well as the Logic Flow paradigm ([7]), consists of several processors, each of them being placed in a node of a graph, which are able to handle data associated with the respective node. Each node processor modifies the local data in accordance with some predefined rules and then the modified data becomes a mobile agent and is relocated in (or, in other words, communicated to) another node of the network, following a given protocol. Only that data which is able to pass a filtering process can be communicated between nodes. The filtering process may require the data to satisfy some conditions imposed by the sending processor, by the receiving processor, or by the edge existing between these nodes (seen as a channel of communication). All the nodes send simultaneously their data and the receiving nodes handle also simultaneously all the arriving messages, according to some strategies, see [8, 11].

Following these lines, in [6] were defined the networks of language processors, as a formal languages generating model. Similarly to the above, such a network can be viewed as a graph whose nodes contain sets of productions and, at any moment of time, a language is associated with a node. In a derivation step, any node derives from its language all possible words as its new language. In a communication step, any node sends those words that satisfy an output condition, given as a regular language, to other nodes, and any node takes those words sent by

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the other nodes, that satisfy an input condition, also given by a regular language. The language generated by a network of language processors consists of all (terminal) words which occur in the languages associated with a given node. Also, having a biological motivation, in [5] one considers a computing model inspired by the evolution of cell populations, which might model some properties of evolving cell communities at the syntactical level. Cells are represented by words which describe their DNA sequences. Informally, at any moment of time, the evolutionary system is described by a collection of words, where each word represents one cell. Cells belong to species and their community evolves according to mutations and division which are defined by operations on words. Only those cells are accepted as surviving (correct) ones which are represented by a word in a given set of words, called the genotype space of the species. This feature parallels with the natural process of evolution. Similar ideas may be met in other bio-inspired models as *tissue-like membrane systems* ([13]) or models from Distributed Computing area like *parallel communicating grammar systems* ([14]).

In this context, in [4], the Networks of Evolutionary Processors (NEPs for short) were defined. More precisely, one considers that each node of a network may be viewed as a cell having genetic information encoded in DNA sequences which may evolve by local evolutionary events, that is point mutations (substitution, deletion or insertion of a symbol). Each node is specialized in just one of these evolutionary operations. Furthermore, the data in each node is organized in the form of multisets of words, each word appearing in an arbitrarily large number of copies, and all the copies are processed in a massive parallel manner, such that all the possible events that can take place do actually take place. Clearly, this computational process is not exactly an evolutionary process in the Darwinian sense. But the rewriting operations considered might be viewed as mutations and the filtering process might be viewed as a selection process. Recombination is missing but it was asserted that evolutionary and functional relationships between genes can be captured by taking only local mutations into consideration ([17]). The computation of a NEP is conducted just as in the case of networks of language processors: initially, the nodes contain some finite sets of words, and further, these words are processed according to the rules in each node, and then, they are communicated to the other nodes, as permitted by the filters associated with the nodes, and so on; the language generated by a NEP consists of all the words which appear in a given node, called the output node.

Results on NEPs, seen as generating devices, can be found, e. g., in [12, 4, 2, 1, 3]. In the seminal paper [4], it was shown that networks of evolutionary processors are universal in that sense that they can generate any recursively enumerable language, and that networks with six nodes and regular filters are sufficient to get all recursively enumerable languages; the result was further improved from the descriptive complexity point of view (see, e. g., [1]) and NEPs having only two types of nodes, and their computational power, were investigated in [2]. Among the same lines, we approach in this paper several variants of NEPs where the communication protocols are restricted.

It is clear that the communication between the nodes of the network depends on two things: the shape of the network (i. e., it depends on which edges exist in the network) and the filters of the nodes. Here we discuss in detail one of these two aspects. Namely, we analyze the computational power of NEPs having the filters of all the nodes in different classes of subregular languages. In [4], and several other papers discussing NEPs as generating devices (see, e. g., [2] and [12] for an early survey), regular filters were used and the computational completeness of the model was shown. Therefore, it seems natural to study in which way the computational

power of the model is affected when simpler filters are used. To summarize, we show that the use of ordered, non-counting, power-separating, circular, and suffix-closed regular languages or languages that are combinational or reverse combinational is as powerful as the use of arbitrary regular languages; on the other hand, the use of only finite languages allows only the generation of regular languages, but not all regular languages can be generated. If only filters are used that are target sets of monoids then non-context-free languages can be generated but not all regular languages.

2. Definitions

We assume that the reader is familiar with the basic concepts of formal language theory (see e. g. [15]). We here only recall some notations used in the paper.

By V^* we denote the set of all words (strings) over V (including the empty word λ). The length of a word w is denoted by $|w|$. The number of occurrences of a letter a or of letters from a set A is denoted by $|w|_a$ and $|w|_A$, respectively. For the number of elements of a set A , we write $\#(A)$.

In the proofs we shall often add new letters of an alphabet U to a given alphabet V . In all these situations, we assume that $V \cap U = \emptyset$.

A phrase structure grammar is specified as a quadruple

$$G = (N, T, P, S)$$

where N is a set of non-terminals, T is a set of terminals, P is a finite set of productions which are written as $\alpha \rightarrow \beta$ with $\alpha \in (N \cup T)^+ \setminus T^*$ and $\beta \in (N \cup T)^*$, and $S \in N$ is the axiom.

A grammar $G = (N, T, P, S)$ is in Geffert normal form ([9]) if the set of nonterminals consists of the axiom S and three more letters A, B, C only and all rules in P have the form

$$ABC \rightarrow \lambda \quad \text{or} \quad S \rightarrow v \text{ with } v \in (N \cup T)^*.$$

By *FIN*, *REG*, *CF*, *CS*, and *RE* we denote the families of finite, regular, context-free, context-sensitive, and recursively enumerable languages, respectively. For a language L over V , we set

$$\begin{aligned} \text{Suf}(L) &= \{ y \mid xy \in L \text{ for some } x \in V^* \}, \\ \text{Comm}(L) &= \{ a_{i_1} \dots a_{i_n} \mid a_1 \dots a_n \in L, n \geq 1, \{i_1, i_2, \dots, i_n\} = \{1, 2, \dots, n\} \}, \\ \text{Circ}(L) &= \{ a_{i+1}a_{i+2} \dots a_na_1a_2 \dots a_i \mid n \geq 1, 1 \leq i \leq n, a_1a_2 \dots a_n \in L \}. \end{aligned}$$

It is known that $\text{Suf}(L)$ and $\text{Circ}(L)$ are regular for a regular language L .

We consider the following restrictions for regular languages. Let L be a language and $V = \text{alph}(L)$ the minimal alphabet of L . We say that L is

- *combinational* iff it can be represented in the form $L = V^*A$ for some subset $A \subseteq V$,
- *bi-combinational* iff it can be represented in the form $L = V^*A$ or $L = AV^*$ for some subset $A \subseteq V$,
- *definite* iff it can be represented in the form $L = A \cup V^*B$ where A and B are finite subsets of V^* ,

- *nilpotent* iff L is finite or $V^* \setminus L$ is finite,
- *commutative* iff $L = \text{Comm}(L)$,
- *circular* iff $L = \text{Circ}(L)$,
- *suffix-closed* (or *fully initial* or *multiple-entry* language) iff $xy \in L$ for some $x, y \in V^*$ implies $y \in L$ (or equivalently, $\text{Suf}(L) = L$),
- *non-counting* (or *star-free*) iff there is an integer $k \geq 1$ such that, for any $x, y, z \in V^*$, $xy^kz \in L$ if and only if $xy^{k+1}z \in L$,
- *power-separating* iff for any $x \in V^*$ there is a natural number $m \geq 1$ such that either $J_x^m \cap L = \emptyset$ or $J_x^m \subseteq L$ where $J_x^m = \{x^n \mid n \geq m\}$,
- *ordered* iff L is accepted by some finite automaton $\mathcal{A} = (Z, V, \delta, z_0, F)$ where $(Z, \preceq)$ is a totally ordered set and, for any $a \in V$, $z \preceq z'$ implies $\delta(z, a) \preceq \delta(z', a)$.

It is obvious that combinational, definite, nilpotent, and ordered languages are regular, whereas non-regular languages of the other types mentioned above exist.

By *COMB*, *BCOMB*, *DEF*, *NIL*, *COMM*, *CIRC*, *SUF*, *NC*, *PS*, and *ORD* we denote the families of all combinational, definite, nilpotent, regular commutative, regular circular, regular suffix-closed, regular non-counting, regular power-separating, and ordered languages, respectively. The relations between these language families are investigated e. g. in [10] and [18] and can be given by Figure 1. Moreover, we add the family *MON* of all languages of the form V^* , where V is an alphabet (languages of *MON* are target sets of monoids; we call them monoidal languages).

Figure 1: Hierarchy of subregular languages (an arrow from X to Y denotes $X \subset Y$ (X is a proper subset of Y), and if two families are not connected by a directed path then they are incomparable)

Let $V = \{a_1, a_2, \dots, a_n\}$ be an alphabet with a fixed order and $\psi: V^* \rightarrow \mathbb{N}_0^n$ be the Parikh mapping with $\psi(w) = (|w|_{a_1}, |w|_{a_2}, \dots, |w|_{a_n})$. The mapping ψ is extended to languages such that the Parikh image of a language L is $\psi(L) = \{\psi(w) \mid w \in L\}$. Two languages L and L' are called letter-equivalent if $\psi(L) = \psi(L')$ (if for any word from one language there is a word in the other language which has the same number of each letter of the alphabet). By ψ^{-1} , we denote the mapping that assigns to a tuple $p = (k_1, k_2, \dots, k_n) \in \mathbb{N}_0^n$ the set of all words w with p as the Parikh vector:

$$\psi^{-1}(p) = \{w \mid |w|_{a_1} = k_1, |w|_{a_2} = k_2, \dots, |w|_{a_n} = k_n\}.$$

The extension to sets of n -tuples yields, for a set M , the set $\psi^{-1}(M) = \{w \mid \psi(w) \in M\}$.

For a language L , we have $\psi(L) = \psi(\text{Comm}(L))$ and $\psi^{-1}(\psi(L)) = \text{Comm}(L)$.

We call a production $\alpha \rightarrow \beta$ a

- substitution if $|\alpha| = |\beta| = 1$,
- deletion if $|\alpha| = 1$ and $\beta = \lambda$.

We introduce insertion as a counterpart of deletion. We write $\lambda \rightarrow a$, where a is a letter. The application of an insertion $\lambda \rightarrow a$ derives from a word w any word w_1aw_2 with $w = w_1w_2$ for some (possibly empty) words w_1 and w_2 .

The productions are applied like context-free rewriting rules. We say that a word v derives a word w , written as $v \Rightarrow w$, if there are words x, y and a production $\alpha \rightarrow \beta$ such that $v = x\alpha y$ and $w = x\beta y$. If the rule p applied is important, we write $v \Rightarrow_p w$.

We now introduce the basic concept of this paper, the networks of evolutionary processors (NEPs for short).

Definition 2.1 *Let X be a family of regular languages.*

- (i) *A network of evolutionary processors (of size n) with filters of the set X is a tuple*

$$\mathcal{N} = (V, N_1, N_2, \dots, N_n, E, j)$$

where

- V is a finite alphabet,
 - for $1 \leq i \leq n$, $N_i = (M_i, A_i, I_i, O_i)$ where
 - M_i is a set of evolutionary rules of a certain type, i. e.,
 $M_i \subseteq \{a \rightarrow b \mid a, b \in V\}$, $M_i \subseteq \{a \rightarrow \lambda \mid a \in V\}$, or $M_i \subseteq \{\lambda \rightarrow b \mid b \in V\}$,
 - A_i is a finite subset of V^* ,
 - I_i and O_i are languages from X over V ,
 - E is a subset of $\{1, 2, \dots, n\} \times \{1, 2, \dots, n\}$, and
 - j is a natural number such that $1 \leq j \leq n$.
- (ii) *A configuration C of $\mathcal{N}$ is an n -tuple $C = (C(1), C(2), \dots, C(n))$ where $C(i)$ is a subset of V^* for $1 \leq i \leq n$.*
- (iii) *Let $C = (C(1), C(2), \dots, C(n))$ and $C' = (C'(1), C'(2), \dots, C'(n))$ be two configurations of $\mathcal{N}$. We say that C derives C' in one*
- evolutionary step (written as $C \Rightarrow C'$) if, for $1 \leq i \leq n$, $C'(i)$ consists of all words $w \in C(i)$ to which no rule of M_i is applicable and of all words w for which there are a word $v \in C(i)$ and a rule $p \in M_i$ such that $v \Rightarrow_p w$ holds,
 - communication step (written as $C \vdash C'$) if, for $1 \leq i \leq n$,

$$C'(i) = (C(i) \setminus O_i) \cup \bigcup_{(k,i) \in E} (C(k) \cap O_k \cap I_i).$$

The computation of $\mathcal{N}$ is a sequence of configurations $C_t = (C_t(1), C_t(2), \dots, C_t(n))$, $t \geq 0$, such that

- $C_0 = (A_1, A_2, \dots, A_n)$,
- for any $t \geq 0$, C_{2t} derives C_{2t+1} in one evolutionary step: $C_{2t} \Rightarrow C_{2t+1}$,
- for any $t \geq 0$, C_{2t+1} derives C_{2t+2} in one communication step: $C_{2t+1} \vdash C_{2t+2}$.

(iv) The language $L(\mathcal{N})$ generated by $\mathcal{N}$ is defined as

$$L(\mathcal{N}) = \bigcup_{t \geq 0} C_t(j)$$

where $C_t = (C_t(1), C_t(2), \dots, C_t(n))$, $t \geq 0$ is the computation of $\mathcal{N}$.

Intuitively, a network with evolutionary processors is a graph consisting of some, say n , nodes $N_1, N_2, \dots, N_n$ (called processors) and the set of edges given by E such that there is a directed edge from N_k to N_i if and only if $(k, i) \in E$. Any processor N_i consists of a set of evolutionary rules M_i , a set of words A_i , an input filter I_i and an output filter O_i . We say that N_i is a substitution node or a deletion node or an insertion node if $M_i \subseteq \{a \rightarrow b \mid a, b \in V\}$ or $M_i \subseteq \{a \rightarrow \lambda \mid a \in V\}$ or $M_i \subseteq \{\lambda \rightarrow b \mid b \in V\}$, respectively. The input filter I_i and the output filter O_i control the words which are allowed to enter and to leave the node, respectively. With any node N_i and any time moment $t \geq 0$ we associate a set $C_t(i)$ of words (the words contained in the node at time t). Initially, N_i contains the words of A_i . In an evolutionary step, we derive from $C_t(i)$ all words applying rules from the set M_i . In a communication step, any processor N_i sends out all words $C_t(i) \cap O_i$ (which pass the output filter) to all processors to which a directed edge exists (only the words from $C_t(i) \setminus O_i$ remain in the set associated with N_i) and, moreover, it receives from any processor N_k such that there is an edge from N_k to N_i all words sent by N_k and passing the input filter I_i of N_i , i.e., the processor N_i gets in addition all words of $C_t(k) \cap O_k \cap I_i$. We start with an evolutionary step and then communication steps and evolutionary steps are alternately performed. The language consists of all words which are in the node N_j (also called the output node, j is chosen in advance) at some moment t , $t \geq 0$.

For a family $X \subseteq REG$, we denote the family of languages generated by networks with evolutionary processors where all filters are of type X by $\mathcal{E}(X)$.

The following fact is obvious.

Lemma 2.2 *Let X, Y be subfamilies of REG such that $X \subseteq Y$. Then $\mathcal{E}(X) \subseteq \mathcal{E}(Y)$.* $\square$

The following theorem is known (see, e. g., [4]).

Theorem 2.3 $\mathcal{E}(REG) = RE$.

3. Results

Let $\mathcal{N} = (V, N_1, N_2, \dots, N_n, E, j)$ be a network with evolutionary processors. In some cases, it is useful to require that the output node N_j has the properties $A_j = \emptyset$, $M_j = \emptyset$, and there is no edge leaving N_j . If $O_j = \emptyset$ then a possibly leaving edge can be removed without further effect. With these properties, a word belongs to the generated language $L(\mathcal{N})$ if and only if it is communicated at some time to N_j . The node N_j itself does not contribute to the whole language, neither by its own initial language nor by modifying words. Additionally, words that have arrived in N_j will not be recycled. We now show that, for each NEP, there is an equivalent NEP with this property.

Theorem 3.1 *For every network $\mathcal{N}$ of evolutionary processors, there is a network $\mathcal{N}'$ of evolutionary processors that generates the same language as $\mathcal{N}$ and has the property that its output node N' has the form $N' = (\emptyset, \emptyset, I', O')$ for some regular languages I', O' over the network's working alphabet and no edge is leaving N' .*

Proof. Let $\mathcal{N} = (V, N_1, N_2, \dots, N_n, E, j)$ be a network of evolutionary processors where the output node N_j has not the required property: $N_j \neq (\emptyset, \emptyset, I_j, O_j)$ for any sets I_j, O_j or there is an edge leaving node N_j . We define a new network $\mathcal{N}' = (V, N'_1, N'_2, \dots, N'_{n+4}, E', n+4)$ by

$$\begin{aligned} N'_i &= N_i \text{ for } 1 \leq i \leq n, \\ N'_i &= (M_i, \emptyset, I_i, O_i) \text{ for } n+1 \leq i \leq n+4, \\ E' &= E \cup \{ (i, n+1) \mid (i, j) \in E \} \\ &\quad \cup \{ (n+1, n+2), (n+1, n+4), (n+2, n+3), (n+2, n+4), (n+3, n+2) \} \end{aligned}$$

where

$$\begin{array}{llll} M_{n+1} = \emptyset, & M_{n+2} = M_j, & M_{n+3} = \emptyset, & M_{n+4} = \emptyset, \\ A_{n+1} = A_j, & A_{n+2} = \emptyset, & A_{n+3} = \emptyset, & A_{n+4} = \emptyset, \\ I_{n+1} = I_j, & I_{n+2} = V^*, & I_{n+3} = V^* \setminus O_j, & I_{n+4} = V^*, \\ O_{n+1} = V^*, & O_{n+2} = V^*, & O_{n+3} = V^*, & O_{n+4} = V^*. \end{array}$$

The network is illustrated below:

The new output node N'_{n+4} satisfies the condition because $N'_{n+4} = (\emptyset, \emptyset, V^*, V^*)$ and no edge leaves the node N'_{n+4} . We now show that $L(\mathcal{N}') = L(\mathcal{N})$.

The subnetwork consisting of $N'_1, N'_2, \dots, N'_n$ is the same as $\mathcal{N}$. The initial sets of N'_j and N'_{n+1} as well as the input filters and incoming edges coincide. Hence, if a word w is in N_j at an even moment t , then w is also in this moment in node N'_j and N'_{n+1} . The word is then sent unchanged to the output node N'_{n+4} . Thus, $w \in L(\mathcal{N})$ and $w \in L(\mathcal{N}')$. Additionally, w is also sent to N'_{n+2} where the same rules as in N_j can be applied. Hence, if a word v is derived in N_j (and, hence, $v \in L(\mathcal{N})$) then v is derived in N'_{n+2} and will be sent to the output node in the next communication step, hence, $v \in L(\mathcal{N}')$. If the word v remains in N_j then a word $u \in L(\mathcal{N})$ will be derived from v in N_j . In $\mathcal{N}'$, the word v will also be sent to N'_{n+3} which takes the word and sends it back to N'_{n+2} where it will be derived to u which will be sent to the output node afterwards. Hence, as long as a word is modified in N_j , the same word is modified in N'_{n+2} with intermediate communication to N'_{n+3} and all these words also arrive in the output node. Thus, $L(\mathcal{N}) \subseteq L(\mathcal{N}')$.

Every word $w \in L(\mathcal{N}')$ came to node N'_{n+4} from node N'_{n+1} or N'_{n+2} . If it came from N'_{n+1} then the word was also in node N_j , hence, $w \in L(\mathcal{N})$. If it came from N'_{n+2} then it has been

derived from a word v which came from N'_{n+1} or N'_{n+3} . If v came from N'_{n+1} then v was also in N_j and has derived w , hence, $w \in L(\mathcal{N})$. If v came from N'_{n+3} then v was previously in node N'_{n+2} and was derived from a word u . Furthermore, $v \notin O_j$. If u came from N'_{n+1} then u was also in N_j and has derived v which remained there and derived w , hence, $w \in L(\mathcal{N})$. If u came from N'_{n+3} then the argumentation can be repeated because for every word in u in N'_{n+2} there was a word $\tilde{u}$ in N'_{n+1} with $\tilde{u} \xRightarrow{*}_{M_j} u$ and all words during this derivation did not belong to O_j . Hence, $\tilde{u}$ was also in N_j where the same derivation of u took place. Thus, we have the inclusion $L(\mathcal{N}') \subseteq L(\mathcal{N})$.

Since $L(\mathcal{N}') = L(\mathcal{N})$ the network $\mathcal{N}'$ has the required properties. $\square$

Networks of evolutionary processors with regular filters that are suffix-closed or circular can generate all recursively enumerable languages.

Theorem 3.2 $\mathcal{E}(SUF) = RE$.

Proof. Let L be a recursively enumerable language. Let $\mathcal{N} = (V, N_1, N_2, \dots, N_n, E, j)$ be a network with evolutionary processors and filters from REG such that $L(\mathcal{N}) = L$. For any node $N_i = (M_i, A_i, I_i, O_i)$, we construct the sets

$$\begin{aligned} I'_i &= \{X\}I_i\{Y\} \cup \text{Suf}(I_i)\{Y\} \cup \{\lambda\}, \\ O'_i &= \{X\}O_i\{Y\} \cup \text{Suf}(O_i)\{Y\} \cup \{\lambda\}, \end{aligned}$$

where X and Y are two new symbols. By definition, I'_i and O'_i are suffix-closed. We assume that the network $\mathcal{N}$ has the property $N_j = (\emptyset, \emptyset, I_j, O_j)$ and no edge leaves the output node (cf. Theorem 3.1).

We consider the network

$$\mathcal{N}' = (V \cup \{X, Y\}, N'_1, N'_2, \dots, N'_n, N'_{n+1}, N'_{n+2}, E', n+2)$$

with

$$\begin{aligned} N'_i &= (M_i, \{X\}A_i\{Y\}, I'_i, O'_i) \text{ for } 1 \leq i \leq n, \\ N'_{n+1} &= (\{X \rightarrow \lambda, Y \rightarrow \lambda\}, \emptyset, I'_j, V^*), \\ N'_{n+2} &= (\emptyset, \emptyset, V^*, \emptyset), \\ E' &= E \cup \{(i, n+1) \mid (i, j) \in E\} \cup \{(n+1, n+2)\}. \end{aligned}$$

It is obvious that the filters of N'_{n+1} and N'_{n+2} are suffix-closed, too. Thus $\mathcal{N}'$ is a network of type SUF .

We now prove that $L(\mathcal{N}) = L(\mathcal{N}')$. We start with words of the form XwY and as long as these words are changed according to rules of M_i , $1 \leq i \leq n$, they can only be sent to nodes N'_s with $1 \leq s \leq n$ and N'_{n+1} . Thus we simulate a derivation in $\mathcal{N}$ (in $\mathcal{N}'$ we have an X in front of and a Y behind the word w occurring in $\mathcal{N}$) and get into N'_{n+1} exactly those words XwY whose subword w comes into N_j . Now X and Y are removed and the resulting word w is sent to N'_{n+2} . Other words cannot arrive in N'_{n+2} and other words do not appear in N_j . Hence, we have the equality $L(\mathcal{N}') = L(\mathcal{N})$. $\square$

Theorem 3.3 $\mathcal{E}(\text{CIRC}) = \text{RE}$.

Proof. We repeat the proof of Theorem 3.2 with the following modifications. We set

$$I'_i = \text{Circ}(\{X\}I_i\{Y\}) \text{ and } O'_i = \text{Circ}(\{X\}O_i\{Y\}) \text{ for } 1 \leq i \leq n.$$

This ensures that $\text{Circ}(F) = F$ for all filters F of the new network $\mathcal{N}'$. Then the proof proceeds as in the case of suffix-closed filters. $\square$

The smallest subfamilies of the family of regular languages we consider are the family of the finite languages (FIN) and the family of all languages of the form V^* for an alphabet V (MON). Every such language can be generated by a NEP where all filters belong to one class given in Figure 1.

Theorem 3.4 For all families X from Figure 1, the inclusion $\text{FIN} \subset \mathcal{E}(X)$ holds.

Proof. According to Lemma 2.2, we have, for all families X from Figure 1, the inclusion

$$\mathcal{E}(\text{FIN}) \subseteq \mathcal{E}(X) \quad \text{or} \quad \mathcal{E}(\text{MON}) \subseteq \mathcal{E}(X).$$

Hence, we only have to show that $\text{FIN} \subset \mathcal{E}(\text{FIN})$ and $\text{FIN} \subset \mathcal{E}(\text{MON})$. Then the theorem follows from these inclusions.

Let L be a finite set over V . Then the networks

$$\mathcal{N}_1^{\text{FIN}} = (V, (\emptyset, L, \emptyset, \emptyset), \emptyset, 1) \quad \text{and} \quad \mathcal{N}_1^{\text{MON}} = (V, (\emptyset, L, \emptyset^*, \emptyset^*), \emptyset, 1)$$

with all filters from either FIN or MON generate L . Thus, any finite language belongs to $\mathcal{E}(\text{FIN})$ and $\mathcal{E}(\text{MON})$.

The networks

$$\mathcal{N}_2^{\text{FIN}} = (\{a\}, (\{\lambda \rightarrow a\}, \{\lambda\}, \emptyset, \emptyset), \emptyset, 1) \quad \text{and} \quad \mathcal{N}_2^{\text{MON}} = (\{a\}, (\{\lambda \rightarrow a\}, \{\lambda\}, \emptyset^*, \emptyset^*), \emptyset, 1)$$

with filters from FIN or MON generate the infinite language $\{a\}^*$. Hence, FIN is a proper subset of $\mathcal{E}(X)$ for $X \in \{\text{FIN}, \text{MON}\}$. $\square$

Theorem 3.5 For all families X from Figure 1, the inclusion $\text{MON} \subset \mathcal{E}(X)$ holds.

Proof. According to Lemma 2.2, we have, for all families X from Figure 1, the inclusion

$$\mathcal{E}(\text{FIN}) \subseteq \mathcal{E}(X) \quad \text{or} \quad \mathcal{E}(\text{MON}) \subseteq \mathcal{E}(X).$$

Hence, we only have to show that $\text{MON} \subset \mathcal{E}(\text{FIN})$ and $\text{MON} \subset \mathcal{E}(\text{MON})$. Then the theorem follows from these inclusions.

Let V be an alphabet and $L = V^*$. Then the networks

$$\begin{aligned} \mathcal{N}_1^{\text{FIN}} &= (V, (\{\lambda \rightarrow a \mid a \in V\}, \{\lambda\}, \emptyset, \emptyset), \emptyset, 1) \text{ and} \\ \mathcal{N}_1^{\text{MON}} &= (V, (\{\lambda \rightarrow a \mid a \in V\}, \{\lambda\}, \emptyset^*, \emptyset^*), \emptyset, 1) \end{aligned}$$

with all filters from either FIN or MON generate L . Thus, any language $L = V^*$ belongs to $\mathcal{E}(\text{FIN})$ and $\mathcal{E}(\text{MON})$.

According to Theorem 3.4, $\mathcal{E}(FIN)$ also contains all finite languages. They cannot be expressed in the form V^* . Hence, $MON \subset \mathcal{E}(FIN)$.

The network

$$\mathcal{N}_2 = (\{a, b\}, (\{\lambda \rightarrow b\}, \{a\}, \emptyset^*, \emptyset^*), \emptyset, 1)$$

generates the language of all words over $\{a, b\}$ with exactly one occurrence of a which is not expressible as V^* . Hence, $MON \subset \mathcal{E}(MON)$. $\square$

We now show that also in more general cases the inclusion

$$X \subseteq \mathcal{E}(X)$$

holds.

Theorem 3.6 *Let X be a class mentioned in Figure 1 except FIN . Each language $L \in X$ can be generated by a NEP $\mathcal{N}$ with exactly two nodes and with filters from type X only.*

Proof. Let X be a class mentioned in Figure 1 except FIN . Then $MON \subseteq X$. Furthermore, let $L \in X$ be a language over an alphabet V . We construct a NEP

$$\mathcal{N} = (V, N_1, N_2, \{(1, 1), (1, 2)\}, 2)$$

with $N_1 = (M_1, A_1, I_1, O_1)$ and $N_2 = (M_2, A_2, I_2, O_2)$ which is defined as follows:

Every word $w \in V^+$ will be derived in node N_1 and be communicated to node N_2 which accepts all words that also belong to L . The language generated by $\mathcal{N}$ is $L(\mathcal{N}) = A_2 \cup (V^+ \cap L) = L$. All filters are of type X . $\square$

If one restricts the networks to be without loops (as usual in the literature, e. g., [1]), the previous statement also holds (just insert another node that only forwards the words it receives from N_1 to N_2).

Together with Theorem 3.4, the previous theorem yields the following statement.

Corollary 3.7 *For each class X mentioned in Figure 1, we have $X \subseteq \mathcal{E}(X)$.*

In the following theorems, we give some results on the set $\mathcal{E}(FIN)$.

Theorem 3.8 $\mathcal{E}(\text{FIN}) \subseteq \text{REG}$.

Proof. Let $\mathcal{N} = (V, N_1, N_2, \dots, N_n, E, j)$ be a network with finite filters. Obviously, a word w is in N_j if and only if it is in A_j or satisfies I_j or is obtained from a word in N_j by application of a rule in M_j . We set

$$U = \{a \mid \lambda \rightarrow a \in M_j\}, \quad V' = \{a' \mid a \in V\}, \quad \text{and} \quad U' = \{a' \mid a \in U\}.$$

Let $h : (V \cup V')^* \rightarrow V^*$ be the homomorphism defined by

$$h(a) = a \text{ for } a \in V \quad \text{and} \quad h(a') = \begin{cases} \lambda, & \text{for } a' \in U', \\ a, & \text{for } a' \in V' \setminus U', \end{cases}$$

and $\tau : (V \cup V')^* \rightarrow V^*$ be the finite substitution where $\tau(a) = \tau(a')$ for $a \in V$ and $\tau(a)$ consists of all $b \in V \cup \{\lambda\}$ such that there are an integers $s \geq 0$ and $b_0, b_1, \dots, b_{s-1} \in V$ and $b_s \in V \cup \{\lambda\}$ such that $a = b_0$, $b = b_s$, and $b_i \rightarrow b_{i+1} \in M_j$ for $0 \leq i \leq s-1$ (note that $s = 0$ implies $a = b$). Furthermore, let

$$k = \max \{|w| \mid w \in O_j \cup I_j \cup A_j\} + 1.$$

We note the following facts:

- Assume that there is a word w of length at least k in $L(\mathcal{N})$. Then w is in $C_t(j)$ for some t . By its length, it cannot leave the node, and thus all words which have a length at least k and can be obtained by application of rules of M_j to w belong to $L(\mathcal{N})$, too.
- If w with $|w| \geq k+1$ is in $L(\mathcal{N})$, then w is obtained from a word $v \in L(\mathcal{N})$ of length k by application of rules in M_j (since substitutions and deletions do not increase the length, the shortest words in $L(\mathcal{N})$ with length at least k are obtained by an insertion from a word of length less than k and thus they have length k).

Now it is easy to see that

$$L(\mathcal{N}) = (L(\mathcal{N}) \cap \bigcup_{i=0}^{k-1} V^i) \cup (\tau(h^{-1}(L(\mathcal{N}) \cap V^k)) \cap \bigcup_{i \geq k} V^i)$$

holds. Since finite languages are regular and regular languages are closed under inverse homomorphisms, finite substitutions, intersection, and union, $L(\mathcal{N})$ is regular. Hence $\mathcal{E}(\text{FIN}) \subseteq \text{REG}$ holds.

Let $L = \{a^n b^m \mid n \geq 1, m \geq 1\}$. Assume that a network $\mathcal{N} = (V, N_1, N_2, \dots, N_n, E, j)$ with finite filters exists such that $L(\mathcal{N}) = L$. Let $k = \max \{|z| \mid z \in O_j \cup I_j \cup A_j\}$ and $w = a^r b^s$ for some $r, s \geq k$. Let $w \in C_t(j)$. By definition of k , the word w is not in I_j and A_j . Thus there is a word $v \in C_{t-1}(j)$ with $v \Rightarrow w$ by application of a rule p of M_j . We distinguish some cases.

Case 1. $p = \lambda \rightarrow a$. Then $v = a^{r-1} b^s$. The word v can also be derived to $a^{r-1} b^s a \in C_t(j)$ and therefore $a^{r-1} b^s a \in L(\mathcal{N})$. However, $a^{r-1} b^s a \notin L$. Hence, we have a contradiction.

Case 2. $p = \lambda \rightarrow b$. Then $v = a^r b^{s-1}$ and $a^r b^{s-1} \Rightarrow b a^r b^{s-1} \in C_t(j) \subseteq L(\mathcal{N}) = L$ in contrast to $b a^r b^{s-1} \notin L$ by definition of L .

Case 3. $p = X \rightarrow x$ for some $X \in V \setminus \{a, b\}$ and $x \in \{a, b, \lambda\}$. Then $v = v' X v''$ for two words v', v'' and $v \in C_{t-1}(j) \subseteq L(\mathcal{N}) = L$. But $v' X v'' \notin L$ by its definition.

Case 4. $p = a \rightarrow b$. If $v = a^r b^p a b^q$ with $p + q = s - 1$ and $p \geq 1$, we get a contradiction as in

Case 3. If $v = a^{r+1} b^{s-1}$ (the rule is applied to the last occurrence of a to obtain w), we can also derive $b a^r b^{s-1}$ and obtain a contradiction as in Case 2.

Case 5. $p = b \rightarrow a$. As in Case 4, we can obtain a contradiction.

Case 6. $p = a \rightarrow \lambda$. Now we get the derivation

$$w = a^r b^s \Longrightarrow a^{r-1} b^s \Longrightarrow a^{r-2} b^s \Longrightarrow \dots \Longrightarrow b^s.$$

Since all words $a^q b^s$ with $q \geq 1$ are not in O_j by the definition of k and $s \geq k$, the words remain in node N_j if a communication step is done. Therefore $b^s \in C_{t+2r}(j) \subseteq L(\mathcal{N})$ in contrast to $b^s \notin L$.

Case 7. $p = b \rightarrow \lambda$. As in Case 6 we can derive a contradiction.

Therefore, our assumption has to be false, which proves $L \notin \mathcal{E}(FIN)$. Because L is regular, the inclusion $\mathcal{E}(FIN) \subseteq REG$ is proper. $\square$

Lemma 3.9 *For each NEP $\mathcal{N}$ with only finite filters, we can construct a NEP $\mathcal{N}'$ with only one processor and finite filters that generates the same language as $\mathcal{N}$.*

Proof. Let $\mathcal{N} = (V, N_1, N_2, \dots, N_n, E, j)$ be a NEP with finite filters. Let B_j be the set of all words that enter some time the node N_j :

$$\begin{aligned} B_j &= \{ w \in V^* \mid \exists t \exists i : (i, j) \in E \text{ and } w \in C_{2t+1}(i) \cap O_i \cap I_j \} \\ &= \left\{ w \in V^* \mid \exists i : (i, j) \in E \text{ and } w \in O_i \cap I_j \cap \bigcup_{t \geq 0} C_{2t+1}(i) \right\}. \end{aligned}$$

The set B_j is finite ($B_j \subseteq I_j$) and can be computed since the set $L'(i) = \bigcup_{t \geq 0} C_{2t+1}(i)$ is regular (see proof of Theorem 3.8, the equation also holds for $L'(i)$ instead of $L(\mathcal{N})$).

Let $\mathcal{N}' = (V, N'_1, \emptyset, 1)$ the NEP with the processor $N'_1 = (M_j, A_j \cup B_j, \emptyset, O_j)$. Let C and C' be the configurations of $\mathcal{N}$ and $\mathcal{N}'$, respectively. We show inductively that every word w which is in node N_j at a time $2t'$ or $2t' + 1$ (for a $t' \geq 0$) is also in node N'_1 at a time $2t''$ or $2t'' + 1$ (for a $t'' \geq 0$), respectively, and vice versa.

- $w \in C_0(j)$. Then $w \in A_j$ and therefore $w \in C'_0(1)$.
- $w \in C_{2t'+1}(j)$ with $t' \geq 0$. Then
 - (a) $w \in C_{2t'}(j)$ and M_j is not applicable or
 - (b) there is a word $v \in C_{2t'}(j)$ which yields w ($v \Longrightarrow_{M_j} w$).

In Case (a), we have $w \in C'_{2t''}(1)$ for a $t'' \geq 0$ by induction hypothesis. Since no rule is applicable, we also have $w \in C'_{2t''+1}(1)$. In Case (b), we have $v \in C'_{2t''}(1)$ for a $t'' \geq 0$ by induction hypothesis. Since $v \Longrightarrow_{M_j} w$ it is $w \in C'_{2t''+1}(1)$.

- $w \in C_{2t'}(j)$ with $t' \geq 1$. Then
 - (a) there is k with $1 \leq k \leq n$, $(k, j) \in E$, and $w \in C_{2t'-1}(k) \cap O_k \cap I_j$ or
 - (b) $w \in C_{2t'-1}(j) \setminus O_j$.

In Case (a), we have $w \in B_j$ and therefore $w \in C'_0(1)$. In Case (b), we have $w \in C'_{2t''+1}(1)$ for a $t'' \geq 0$ by induction hypothesis. Since $w \notin O_j$ we also have $w \in C'_{2t''+2}(1)$.

- $w \in C'_0(1)$. Then

- (a) $w \in A_j$ or
- (b) $w \in B_j$.

In Case (a), we also have $w \in C_0(j)$. In Case (b), we have $w \in C_{2t'}(j)$ for a $t' \geq 1$.

- $w \in C'_{2t''+1}(1)$ with $t'' \geq 0$. Then

- (a) $w \in C'_{2t''}(1)$ and M_j is not applicable or
- (b) there is a word $v \in C'_{2t''}(1)$ which yields w ($v \Rightarrow_{M_j} w$).

In Case (a), we have $w \in C_{2t'}(j)$ for a $t' \geq 0$ by induction hypothesis. Since no rule is applicable, we also have $w \in C_{2t'+1}(j)$. In Case (b), we have $v \in C_{2t'}(j)$ for a $t' \geq 0$ by induction hypothesis. Since $v \Rightarrow_{M_j} w$ it is $w \in C_{2t'+1}(j)$.

- $w \in C'_{2t''}(1)$ with $t'' \geq 1$. Then $w \in C'_{2t''-1}(1) \setminus O_j$ (because there is no ‘real’ communication). We have $w \in C_{2t'+1}(j)$ for a $t' \geq 0$ by induction hypothesis. Since $w \notin O_j$ we also have $w \in C_{2t'+2}(j)$.

By this induction, it is shown that $L(\mathcal{N}') = L(\mathcal{N})$. □

Let $\mathcal{N} = (V, N_1, \emptyset, 1)$ be a NEP with only finite filters and $N_1 = (M, A, \emptyset, O)$. If N_1 is a deleting or substituting processor then the language generated is finite ([2]). Let now N_1 be an inserting processor. Furthermore, let $U = \{a \mid \lambda \rightarrow a \in M\}$ be the set of all letters that can be inserted. If $O = \emptyset$ then the language generated is

$$L(\mathcal{N}) = \{u_1x_1u_2x_2 \cdots u_nx_nu_{n+1} \mid x_1x_2 \cdots x_n \in A \text{ and } u_i \in U^* \text{ for } 1 \leq i \leq n+1\}.$$

In the case that $O \neq \emptyset$, let $\min_O = \min\{|w| \mid w \in O\}$, $\max_O = \max\{|w| \mid w \in O\}$, and

$$\begin{aligned} A_1 &= \{w \mid w = u_1x_1u_2x_2 \cdots u_nx_nu_{n+1}, x_1x_2 \cdots x_n \in A \text{ and } u_i \in U^* \\ &\quad \text{for } 1 \leq i \leq n+1, \text{ and } |w| < \min_O\}, \\ A_2 &= \{w \mid w = u_1x_1u_2x_2 \cdots u_nx_nu_{n+1}, x_1x_2 \cdots x_n \in A \text{ and } u_i \in U^* \\ &\quad \text{for } 1 \leq i \leq n+1, \text{ and } |w| = \min_O\}, \\ A_3 &= \{w \mid w = u_1x_1u_2x_2 \cdots u_nx_nu_{n+1}, x_1x_2 \cdots x_n \in A_2 \setminus O \text{ and } u_i \in U^* \\ &\quad \text{for } 1 \leq i \leq n+1, \text{ and } |w| = \min_O + 1\}, \\ A_4 &= \{w \mid w = u_1x_1u_2x_2 \cdots u_nx_nu_{n+1}, x_1x_2 \cdots x_n \in A_3 \setminus O \text{ and } u_i \in U^* \\ &\quad \text{for } 1 \leq i \leq n+1, \text{ and } |w| = \min_O + 2\}, \\ &\vdots \\ A_{\Delta+4} &= \{w \mid w = u_1x_1u_2x_2 \cdots u_nx_nu_{n+1}, x_1x_2 \cdots x_n \in A_{\Delta+3} \setminus O \text{ and } u_i \in U^* \\ &\quad \text{for } 1 \leq i \leq n+1, \text{ and } |w| = \max_O + 1\}, \\ A_{\Delta+5} &= \{w \mid w = u_1x_1u_2x_2 \cdots u_nx_nu_{n+1}, x_1x_2 \cdots x_n \in A_{\Delta+4} \text{ and } u_i \in U^* \\ &\quad \text{for } 1 \leq i \leq n+1, \text{ and } |w| > \max_O + 1\} \end{aligned}$$

with $\Delta = (\max_O + 1) - (\min_O + 2) = \max_O - \min_O - 1$.

Then the language generated is

$$L(\mathcal{N}) = \bigcup_{i=1}^{\Delta+5} A_i.$$

Theorem 3.10 $\mathcal{E}(\text{FIN}) \subset \mathcal{E}(\text{NIL})$.

Proof. The inclusion $\mathcal{E}(\text{FIN}) \subseteq \mathcal{E}(\text{NIL})$ follows from Lemma 2.2.

Let $V = \{a\}$ and $L = \{a\} \cup \{a^n \mid n \geq 3\}$. The complement of L is $\bar{L} = \{\lambda, aa\}$ which is finite. Hence, L is a nilpotent language. By Corollary 3.7, $L \in \mathcal{E}(\text{NIL})$.

Suppose L is generated by a network with only finite filters. Then, by Lemma 3.9, there is a network $\mathcal{N}$ with only one node $N = (M, A, \emptyset, O)$ that generates L . Since L is infinite, this node must be inserting. Hence, the rule set is $M = \{\lambda \rightarrow a\}$. If the initial set A contains λ then $\lambda \in L(\mathcal{N})$ which is in contrast to $\lambda \notin L$. If the initial set A contains a or aa then the word aa belongs to the generated language $L(\mathcal{N})$ which is in contrast to $aa \notin L$. If the initial set only contains words a^n with $n \geq 3$ then the word a is not generatable but $a \in L$ which is a contradiction, too. Hence, there is no network with only finite filters that generates L . Thus, we have $L \in \mathcal{E}(\text{NIL}) \setminus \mathcal{E}(\text{FIN})$. $\square$

Let $\mathcal{N} = (V, N_1, N_2, \dots, N_n, E, j)$ be a network of evolutionary filters where all filters and all initial languages A_i for $1 \leq i \leq n$ are commutative. Then, for each node N_i with $1 \leq i \leq n$ and each time $t \geq 0$, the set $C_t(i)$ is commutative, too. Hence, the generated language $L(\mathcal{N})$ is also commutative.

We next show that only commutative filters are not sufficient to generate all recursively enumerable languages.

Theorem 3.11 $\mathcal{E}(\text{COMM}) \subset \mathcal{E}(\text{CIRC})$.

Proof. The inclusion $\mathcal{E}(\text{COMM}) \subseteq \mathcal{E}(\text{CIRC})$ follows from Lemma 2.2.

Let $L = \{a^n b^n \mid n \geq 1\}$. According to Theorem 3.3, we have $L \in \mathcal{E}(\text{CIRC})$. Let us assume that $L \in \mathcal{E}(\text{COMM})$. Then there is a NEP $\mathcal{N}$ which has only commutative filters and which generates the language L . Since L is infinite and networks with only substitution and deletion nodes always generate finite languages, $\mathcal{N}$ contains an inserting processor. The number of as is unbounded (for each natural number n , there is a word $w \in L$ with more than n occurrences of a). Hence, there are a natural number $s \geq 0$ and letters $x_0, x_1, \dots, x_s$ with $x_s = a$ such that the network contains the rules $\lambda \rightarrow x_0$ and $x_i \rightarrow x_{i+1}$ for $0 \leq i \leq s-1$ and there is a word $w_1 a w_2 \in L$ which is derived from a word $v_1 v_2$ by applying these rules (possibly not only these rules), starting with the insertion of x_0 between v_1 and v_2 . Instead, x_0 could also be inserted at the end of $v_1 v_2$. All words derived from $v_1 x_0 v_2$ are letter equivalent to those derived from $v_1 v_2 x_0$. Thus, if a word derived from $v_1 x_0 v_2$ can pass a filter then also a word that is derived from $v_1 v_2 x_0$ in the same manner can pass that filter. Hence, in the same way how $w_1 a w_2$ is derived and communicated to the output node, also the word $w_1 w_2 a$ is derived and communicated to the output node. But $w_1 w_2 a \notin L$. Thus the language L cannot be generated by a NEP where all filters are commutative. This implies $L \notin \mathcal{E}(\text{COMM})$ and the proper inclusion $\mathcal{E}(\text{COMM}) \subset \mathcal{E}(\text{CIRC})$ is shown. $\square$

We now continue with investigations on the class $\mathcal{E}(\text{MON})$.

Theorem 3.12 *Let L be a semi-linear language. Then $\text{Comm}(L) \in \mathcal{E}(\text{MON})$.*

Proof. For each semi-linear language L , a regular grammar G can be constructed which generates a language that is letter-equivalent to L , i. e., $\psi(L(G)) = \psi(L)$ ([16]). Then we have

$$\psi^{-1}(\psi(L(G))) = \psi^{-1}(\psi(L))$$

and therefore $\text{Comm}(L(G)) = \text{Comm}(L)$. Thus, for any semi-linear language L , a regular grammar G can be constructed with $\text{Comm}(L(G)) = \text{Comm}(L)$. We now give, for a regular grammar G , a network with only monoidal filters that generates the language $\text{Comm}(L(G))$. Then we have a method to construct, for a semi-linear language L , a network with only monoidal filters that generates the language $\text{Comm}(L)$.

Let $G = (N, T, P, S)$ be a regular grammar where all rules in P have the form $A \rightarrow aB$ or $A \rightarrow a$ for nonterminals $A, B \in N$ and a terminal symbol $a \in T$. Additionally, the rule $S \rightarrow \lambda$ is permitted if the axiom S does not occur on the right hand side of a rule.

For each nonterminal $X \in N$, we set

$$R(X) = \{ \langle aY \rangle \mid X \rightarrow aY \in P \}$$

as the set of symbols representing the right hand sides of the nonterminating rules,

$$T_t(X) = \{ a \mid a \in T \text{ and } X \rightarrow a \in P \}$$

as the set of all terminal symbols that are generated by X with terminating,

$$N_{\text{post}}(X) = \{ Y \mid Y \in N \text{ and } \exists a \in T : X \rightarrow aY \in P \}$$

as the set of all nonterminals that are generated by X , and

$$T_{\text{pre}}(X) = \{ a \mid a \in T \text{ and } \exists Y \in N : Y \rightarrow aX \in P \}$$

as the set of all terminal symbols that are generated at the same time as X . The terminating nonterminals (those nonterminals $X \in N$ for which a rule $X \rightarrow a \in P$ exists for a terminal symbol $a \in T$) are gathered in the set N_t . Furthermore, we set $R = \{ \langle aY \rangle \mid \exists X \in N : X \rightarrow aY \in P \}$ and $\hat{T} = T \cup R$.

For every nonterminal $X \in N$, we define two nodes

$$N_X = (M_X, A_X, I_X, O_X) \quad \text{and} \quad N_{X'} = (M_{X'}, A_{X'}, I_{X'}, O_{X'})$$

as follows:

$$\begin{aligned} M_X &= \{ \lambda \rightarrow \langle aY \rangle \mid X \rightarrow aY \in P \}, & M_{X'} &= \{ \langle aX \rangle \rightarrow a \mid a \in T_{\text{pre}}(X) \}, \\ A_X &= \emptyset, & A_{X'} &= \begin{cases} \{ \lambda \}, & \text{if } X = S, \\ \emptyset, & \text{otherwise,} \end{cases} \\ I_X &= T^*, & I_{X'} &= (\{ \langle aX \rangle \mid a \in T_{\text{pre}}(X) \} \cup T)^*, \\ O_X &= \hat{T}^*, & O_{X'} &= T^*. \end{aligned}$$

For each terminating nonterminal $X \in N_t$, we additionally define a node

$$N_{X_t} = (M_{X_t}, A_{X_t}, I_{X_t}, O_{X_t})$$

by $M_{X_t} = \{ \lambda \rightarrow a \mid a \in T_t(X) \}$, $A_{X_t} = \emptyset$, and $I_{X_t} = O_{X_t} = T^*$.

The output node $N_{out} = (M_{out}, A_{out}, I_{out}, O_{out})$ which is responsible for the language generated is defined by $M_{out} = A_{out} = \emptyset$ and $I_{out} = O_{out} = T^*$.

The edges of the network will be as follows: there is an edge from node N_X to node $N_{Y'}$ if $Y \in N_{post}(X)$, an edge from node $N_{X'}$ to node N_X for each nonterminal $X \in N$, an edge from $N_{X'}$ to N_{X_t} for each terminating nonterminal $X \in N_t$, an edge from N_{X_t} to N_{out} for each terminating nonterminal $X \in N_t$, and if the rule $S \rightarrow \lambda$ exists in P then there is an edge from node $N_{S'}$ to node N_{out} :

$$E = \{ (X, Y' \mid Y \in N_{post}(X)) \} \cup \{ (X', X) \mid X \in N \} \cup \{ (X', X_t) \mid X \in N_t \} \\ \cup \{ (X_t, out) \mid X \in N_t \} \cup E_\lambda$$

with

$$E_\lambda = \begin{cases} \{ (S', out) \}, & \text{if } S \rightarrow \lambda \in P, \\ \emptyset, & \text{otherwise.} \end{cases}$$

The network with the working alphabet $\hat{T}$, the nodes described above, the edge set E and the output node N_{out} is denoted by $\mathcal{N}$. Note that all filters belong to the family *MON*.

The network simulates G in the sense that a terminal symbol is created somewhere in a word whenever it is appended by the grammar. The nonterminal in a sentential form gives the ‘state’ in which the sentential form currently is. This is represented in the network by the different nodes. The derivation in G starts with the axiom S . The simulation starts in node $N_{S'}$ (the only node where the initial set is not empty). In this moment, no rule can be applied. So, the empty word is sent to the node N_S and to N_{S_t} if there is a rule $S \rightarrow a \in P$ for a terminal symbol a and to N_{out} if the rule $S \rightarrow \lambda$ exists in P . In the latter case, we have $\lambda \in L(G)$ and $\lambda \in L(\mathcal{N})$.

For each rule $X \rightarrow aY \in P$, the node N_X can generate the intermediate symbol $\langle aY \rangle$. Then the word is sent out to all nodes N_Z (where Z is on the right hand side of a rule for X : $Z \in N_{post}(X)$), also if the only rule with X and Z is $X \rightarrow bZ$), but only the node N_Y takes the word. This node changes the symbol $\langle aY \rangle$ to a and sends the word to the node N_Y and, if Y is a terminating nonterminal, also to the node N_{Y_t} .

Whenever a node N_{X_t} receives a word w then wX is letter-equivalent to a word vX that is a sentential form of G . Now every terminal symbol a that can be appended to v in a terminating derivation step can be inserted into the word w . This word is sent to the output node. Hence, it belongs to the generated language.

Thus, $L(\mathcal{N}) = \{ w \mid w \in T^* \text{ and } \exists v \in L(G) : \psi(v) = \psi(w) \} = \text{Comm}(L(G))$. $\square$

Corollary 3.13 *There are non-context-free languages that can be generated by networks with only filters from the family *MON*.*

Proof. Let n be a natural number, $V = \{ a_1, a_2, \dots, a_n \}$ and L be the non-context-free language $L = \{ w \mid w \in V^*, |w|_{a_1} = |w|_{a_2} = \dots = |w|_{a_n} \}$. This language is semi-linear and commutative. Hence, $L = \text{Comm}(L)$. According to Theorem 3.12, $L \in \mathcal{E}(\text{MON})$. $\square$

Even some languages that are not semi-linear can be generated as the following example shows.

Lemma 3.14 *The non-semi-linear language $L = \{a^{2^n} \mid n \geq 0\}$ can be generated by a network with only monoidal filters.*

Proof. Let $V = \{S, A, F, a\}$ and $\mathcal{N} = (V, N_1, N_2, N_3, N_4, N_5, E, 5)$ be the following network:

In the beginning, we have the word S in node N_1 . We consider a word S^n for $n \geq 1$ in node N_1 in an even moment (in the beginning or after a communication step). One occurrence of S is replaced by A , then the word is sent to node N_2 where another copy of A is inserted. This word w goes back to node N_1 and it goes on to node N_3 which takes it if no S appears in the word. If in N_1 the rule $A \rightarrow F$ is applied then the symbol F is introduced which cannot be replaced. Due to the output filter O_1 , the word will be trapped in N_1 for ever. If in the word w no S is present then the only rule which can be applied is $A \rightarrow F$ and the cycle is stopped. If w still contains an S then it is replaced by A and N_2 inserts another A . So, the words move between N_1 and N_2 where alternately an S is replaced by A and an A is inserted until the word only contains A s. The word is then A^{n+1} . Hence, the number of letters has been doubled.

In N_3 , each A is replaced by S . The word is S^{n+1} when it leaves N_3 . It moves to N_1 and to N_4 . In N_1 , the cycle starts again with a word S^m for $m \geq 1$. All arriving words in N_4 have the form S^m with $n \geq 2$. In order to cover also the case $n = 1$, the initial language of this node consists of S . In N_4 , every letter S is replaced by the symbol a before the word leaves to node and moves to the output node N_5 .

Hence, $L(\mathcal{N}) = \{a^{2^n} \mid n \geq 0\}$. □

An upper bound for the class $\mathcal{E}(\text{MON})$ is given in the following theorem.

Theorem 3.15 $\mathcal{E}(\text{MON}) \subset \mathcal{E}(\text{COMB})$.

Proof. The inclusion $\mathcal{E}(\text{MON}) \subseteq \mathcal{E}(\text{COMB})$ follows from Lemma 2.2.

Let $L = \{wb \mid w \in \{a, b\}^*\}$. The language L can be written as V^*B with $V = \{a, b\}$ and $B = \{b\}$. Hence, $L \in \text{COMB}$ and, by Corollary 3.7, $L \in \mathcal{E}(\text{COMB})$.

Suppose $L \in \mathcal{E}(\text{MON})$. Then there is a NEP $\mathcal{N}$ which has only filters that belong to the class MON and which generates the language L . Since L is infinite and networks with only substitution and deletion nodes generate finite languages, $\mathcal{N}$ contains an inserting processor. The number of a s is unbounded (for each natural number n , there is a word $w \in L$ with more than n occurrences of a). Hence, there are a natural number $s \geq 0$ and letters $x_0, x_1, \dots, x_s$ with $x_s = a$ such that the network contains the rules $\lambda \rightarrow x_0$ and $x_i \rightarrow x_{i+1}$ for $0 \leq i \leq s-1$.

and there is a word $w_1aw_2 \in L$ which is derived from a word v_1v_2 by applying these rules (possibly not only these rules), starting with the insertion of x_0 between v_1 and v_2 . Instead, x_0 could also be inserted at the end of v_1v_2 . All words derived from $v_1x_0v_2$ are letter equivalent to those derived from $v_1v_2x_0$. Thus, if a word derived from $v_1x_0v_2$ can pass a filter then also a word that is derived from $v_1v_2x_0$ in the same manner can pass that filter. Hence, in the same way how w_1aw_2 is derived and communicated to the output node, also the word w_1w_2a is derived and communicated to the output node. But $w_1w_2a \notin L$. Thus, the language L cannot be generated by a NEP where the filters belong to MON . This implies $L \notin \mathcal{E}(MON)$ and therefore $\mathcal{E}(MON) \subset \mathcal{E}(COMB)$. $\square$

Corollary 3.16 *The classes REG of all regular languages and CF of all context-free languages are incomparable with the class $\mathcal{E}(MON)$ of all languages generatable by networks with only monoidal filters.*

Proof. The regular language

$$L = \{wb \mid w \in \{a,b\}^*\}$$

cannot be generated by a network with filters from MON only (according to the proof of Theorem 3.15). However, the non-context-free language

$$L = \{w \mid w \in \{a,b,c\}^*, |w|_a = |w|_b = |w|_c\}$$

can be generated (Corollary 3.13). $\square$

We now further investigate the class $\mathcal{E}(COMB)$.

Theorem 3.17 $REG \subset \mathcal{E}(COMB)$.

Proof. We first show that a regular language can be generated by a NEP with combinational filters only. The idea is based on the network constructed in the proof of Theorem 3.12. Since we now have combinational filters, we can check whether a symbol has been appended to a word (and not only inserted at an arbitrary position). We modify the network $\mathcal{N}$ from the proof of Theorem 3.12 to a network $\mathcal{N}'$ as follows: We introduce a new set $T' = \{a' \mid a \in T\}$ and set $\hat{T} = T \cup R \cup T'$. The input filters $I_{X'}$ are now $I_{X'} = \hat{T}^* \{ \langle aX \rangle \mid a \in T_{pre}(X) \}$ (the change from T to $\hat{T}$ has only formal reasons because it has to be a superset of the set of the last symbols; the network ensures that there is exactly one bracketed symbol in an arriving word). The rules in the nodes N_{X_t} representing the terminating rules are changed to $M_{X_t} = \{ \lambda \rightarrow a' \mid a \in T_t(X) \}$ (because we have to check again whether the inserted symbol was appended). Then these output filters are $O_{X_t} = \hat{T}^*$. For checking whether the last symbol was inserted at the end of the word, we introduce a new node $N_{X'_t}$ (for each $X \in N_t$) with

$$M_{X'_t} = \{ a' \rightarrow a \mid a \in T_t(X) \}, A_{X'_t} = \emptyset, I_{X'_t} = \hat{T}^* \{ a' \mid a \in T_t(X) \}, \text{ and } O_{X'_t} = T^*.$$

This node also replaces the primed symbol by its unprimed version. The network $\mathcal{N}'$ simulates the derivation of the grammar G . Hence, $L(\mathcal{N}') = L(G)$. Thus, $REG \subseteq \mathcal{E}(COMB)$.

Let $L = \{w \mid w \in \{a,b\}^* \text{ and } |w|_a = |w|_b\}$. This language is not regular but, according to Theorem 3.12, $L \in \mathcal{E}(MON)$ (because L is semi-linear and $Comm(L) = L$) and therefore $L \in \mathcal{E}(COMB)$. Thus, $REG \subset \mathcal{E}(COMB)$. $\square$

Lemma 3.18 *The language $L = \{a^n b^n \mid n \geq 1\}$ can be generated by a network with only combinational filters.*

Proof. Let $V = \{a, B, B', B'', b\}$ and $\mathcal{N} = (V \cup \{F\}, N_1, N_2, \dots, N_7, E, 7)$ be the following network:

In the cycle consisting of N_1 and N_2 , words are produced that have the same number of a s and B s. Whenever a word w moves from N_2 to N_3 it satisfies $w \in \{a, B\}^*$ with $|w|_a = |w|_B = n$ where $n \geq 1$. In the cycle consisting of N_3, N_4, N_5, N_6 , a B is deleted and a b is inserted at the end of the word. The word stays in this cycle until all B s are consumed. In the end, the word is $a^n b^n$ in node N_5 . This word enters the output node N_6 and also the node N_3 . But there, only the rule $a \rightarrow F$ can be applied which introduces the trap symbol F and the word remains in this node for ever.

Hence, $L(\mathcal{N}) = L = \{a^n b^n \mid n \geq 1\}$ and $L \in \mathcal{E}(\text{COMB})$. $\square$

This language $L = \{a^n b^n \mid n \geq 1\}$ is a witness of the following proper inclusion.

Theorem 3.19 $\mathcal{E}(\text{NIL}) \subset \mathcal{E}(\text{DEF})$.

Proof. The inclusion $\mathcal{E}(\text{NIL}) \subseteq \mathcal{E}(\text{DEF})$ follows from Lemma 2.2. We now show that it is proper.

Let $L = \{a^n b^n \mid n \geq 1\}$. According to Lemma 3.18, $L \in \mathcal{E}(\text{COMB})$. Hence, also $L \in \mathcal{E}(\text{DEF})$ holds.

Let us assume that $L = L(\mathcal{N})$ for some evolutionary network $\mathcal{N} = (V, N_1, N_2, \dots, N_r, E, j)$ with $N_i = (M_i, A_i, I_i, O_i)$ with a finite set A_i and nilpotent filters I_i and O_i for $1 \leq i \leq r$. Then there is a natural number s such that

$$A_i \subseteq \bigcup_{k=0}^s V^k, \quad I_i \subseteq \bigcup_{k=0}^s V^k \text{ or } I_i = I'_i \cup V^s V^+ \text{ with } I'_i \subseteq \bigcup_{k=0}^s V^k,$$

and

$$O_i \subseteq \bigcup_{k=0}^s V^k \text{ or } O_i = O'_i \cup V^s V^+ \text{ with } O'_i \subseteq \bigcup_{k=0}^s V^k$$

for $1 \leq i \leq r$. Let

$$D = w_0 \Rightarrow_{M_{q(1)}} w_1 \vdash w_1 \Rightarrow_{M_{q(2)}} w_2 \vdash w_2 \Rightarrow_{M_{q(3)}} w_3 \vdash \dots \Rightarrow_{M_{q(p)}} w_p \vdash w_p$$

be a derivation (the evolution steps are done according to the sets $M_{q(l)}$, $1 \leq l \leq p$) and w_p is contained in the output node.

We say that a letter x is an a -occurrence (or b -occurrence or λ -occurrence) in w_t , $1 \leq t \leq p$, if in all steps $w_{t+u} \Rightarrow_{M_{q(t+u+1)}} w_{t+u+1}$, $u \geq 0$, the letter or its successors are not changed or changed by a substitution and the last change is a substitution of the form $z \rightarrow a$ (or $z \rightarrow b$ or $z \rightarrow \lambda$, respectively).

We say that w_t is a Z-word if all steps $w_{t+u} \Rightarrow_{M_{q(t+u+1)}} w_{t+u+1}$, $u \geq 0$, are substitutions or deletions or insertions of λ -occurrences.

Let now $w_p = a^s b^s$. Moreover, let w_l be the first occurrence of a Z-word in D . By the definition of a Z-word, all words $w_l, w_{l+1}, \dots, w_p$ have at least the length $2s$. Thus, only infinite filters are used and if w_{l+u} , $u \geq 0$, passes such a filter, then any permutation of w_{l+u} passes the filter, too. Moreover,

$$w_l = c_1 a_1 c_2 a_2 \dots c_s a_s c_{s+1} b_1 c_{s+2} b_2 \dots c_{2s} b_s c_{2s+1}$$

with several a -occurrences $a_1, a_2, \dots, a_s$, several b -occurrences $b_1, b_2, \dots, b_s$ and several (possibly empty) words $c_1, c_2, \dots, c_{2s+1}$ consisting of λ -occurrences. Furthermore, $w_{l-1} \Rightarrow_{M_{q(l)}} w_l$ is an insertion of an a -occurrence or an insertion of a b -occurrence. Let us assume that a_f or b_g is inserted. Now we change the position of the insertion in the step $w_{l-1} \Rightarrow_{M_{q(l)}} w_l$ such that

$$w_{l-1} \Rightarrow_{M_{q(l)}} c_1 a_1 c_2 a_2 \dots c_{f-1} a_{f-1} c_f c_{f+1} a_{f+1} \dots c_s a_s c_{s+1} b_1 c_{s+2} b_2 \dots c_{2s} b_s a_f c_{2s+1}$$

or

$$w_{l-1} \Rightarrow_{M_{q(l)}} c_1 b_g a_1 c_2 a_2 \dots c_s a_s c_{s+1} b_1 c_{s+2} b_2 \dots c_{s+g-1} b_{g-1} c_{s+g} c_{s+g+1} b_{g+1} \dots c_{2s} b_s a_f c_{2s+1}$$

is obtained. In the sequel, we perform the same operations as in D . Thus, we obtain finally $a^{s-1} b^s a$ or $b a^s b^{s-1}$ in the output node. Therefore

$$a^{s-1} b^s a \in L(\mathcal{N}) \text{ or } b a^s b^{s-1} \in L(\mathcal{N}),$$

but

$$a^{s-1} b^s a \notin L \text{ and } b a^s b^{s-1} \notin L,$$

which contradicts $L = L(\mathcal{N})$.

Thus, $L \notin \mathcal{E}(\text{NIL})$. Hence, we have a language in $\mathcal{E}(\text{DEF}) \setminus \mathcal{E}(\text{NIL})$ and the proper inclusion $\mathcal{E}(\text{NIL}) \subset \mathcal{E}(\text{DEF})$ is shown. $\square$

If the filters are combinational or reverse combinational languages, i. e., languages of the form $V^* A$ or AV^* , then every recursively enumerable language can be generated.

Theorem 3.20 $\mathcal{E}(\text{BCOMB}) = \text{RE}$.

Proof. Let L be a recursively enumerable language and $G = (N, T, P, S)$ be a grammar in Geffert normal form generating the language $L(G) = L$ with $N = \{S, A, B, C\}$. We construct a NEP $\mathcal{N}$ that simulates the derivation process in G and, hence, generates the same language. The idea is to rotate the sentential form until the subword which has to be replaced is a suffix and then to delete and insert (if necessary) at the right end of the word.

Let $V = N \cup T$ and $\# \notin V$ be a symbol that marks the actual end of a word during the rotation. Let $U = V \cup \{\#\}$, $u = \#(U)$ be the number of the elements of U , and $x_1, x_2, \dots, x_u$ be the elements of U . Additionally, let $U' = \{x' \mid x \in U\}$ and $\hat{U} = U \cup U'$. Furthermore, let p be the number of the rules of the form $S \rightarrow v$ and let these rules be $P_i = S \rightarrow a_{i,1}a_{i,2} \dots a_{i,l_i}$ with $1 \leq i \leq p$, $l_i \geq 0$, and $a_{i,j} \in V$ for $1 \leq j \leq l_i$.

We construct subnetworks for rotating a word and for simulating the application of a rule. These subnetworks are connected by a special node (the ‘master’ N_0) that belongs to every subnetwork. The structure of the network can be seen in Figure 2.

Figure 2: NEP for a Geffert normal form

The master N_0 is defined by $A_0 = \{S\#\}$, $M_0 = \emptyset$, $I_0 = U^+$, and $O_0 = U^+$. The work starts with the first sentential form of G where the end is marked by $\#$. This node does nothing but in the next communication step the word will be sent to the nodes $N_{1,rot}$ where the rotation will start and $N_{1,sim}$ where the simulation of a rule will start.

The initial languages A_i of all other nodes N_i are empty.

The node $N_{1,rot}$ is defined by $I_{1,rot} = U^+$, $M_{1,rot} = \{x \rightarrow x' \mid x \in U\}$, and $O_{1,rot} = \hat{U}^+$. This node marks a symbol by changing it to a primed version. Then the word is sent to all nodes $N_{2,x}$ where $x \in U$. If another symbol than the left most one has been marked, the word leaves the network. If the left most symbol has been marked, then exactly one of these nodes gives access to the word, namely the node $N_{2,x}$ where x is the symbol whose primed version stands at the beginning of the word. Thus, $N_{2,x}$ for $x \in U$ is defined by $I_{2,x} = \{x'\}\hat{U}^*$, $M_{2,x} = \{x' \rightarrow \lambda\}$, and $O_{2,x} = U^+$. The primed x is deleted. Then the word is sent to the corresponding node $N_{3,x}$ where x' is inserted. Hence, $N_{3,x}$ for $x \in U$ is defined by $I_{3,x} = U^+$, $M_{3,x} = \{\lambda \rightarrow x'\}$, and $O_{3,x} = \hat{U}^+$. Then the word is sent to node $N_{4,x}$ which takes the word only if x' is the

last symbol (the right most symbol) of the word and replaces x' by x . Hence, $N_{4,x}$ for $x \in U$ is defined by $I_{4,x} = \hat{U}^*\{x'\}$, $M_{4,x} = \{x' \rightarrow x\}$, and $O_{4,x} = U^+$. Now, the word was rotated by one symbol and will be sent to the master.

As mentioned above, the simulation of a rule will start in the node $N_{1,sim}$. It is defined by $I_{1,sim} = U^+$, $M_{1,sim} = \{x \rightarrow x' \mid x \in U\}$, and $O_{1,sim} = \hat{U}^+$. In this node, a symbol will be changed to the primed version. If another symbol than the right most one is primed, the word is lost in the next communication step. If the last symbol was C and the rule $ABC \rightarrow \lambda$ exists in P then this rule can be started to be simulated (maybe C is not preceded by AB then the simulation does not work which will be noticed later). If the last symbol was S then one of the rules P_i ($1 \leq i \leq p$) can be simulated. If the last symbol was $\#$ then this end marker should be removed to obtain a real sentential form of G (which is the end of the simulation of G). In all other cases, we do not need the word anymore.

We now construct subnetworks for these three cases.

If the erasing rule $ABC \rightarrow \lambda$ exists in P then we define the following nodes: $N_{5,C}$ is defined by $I_{5,C} = \hat{U}^*\{C'\}$, $M_{5,C} = \{C' \rightarrow \lambda\}$, and $O_{5,C} = U^+$. This node takes a word if the last symbol is C' , it deletes this symbol and passes on the word to the next node $N_{6,B}$ which is responsible for the B . The node $N_{6,B}$ is defined by $I_{6,C} = U^*\{B\}$, $M_{6,B} = \{B \rightarrow \lambda\}$, and $O_{6,B} = U^+$. This node takes the word if the last symbol is B , otherwise the word is lost because the previously selected C does not belong to a subword ABC and therefore the rule cannot be applied. The node $N_{6,B}$ deletes a B , and sends the word to node $N_{7,A}$ defined by $I_{7,A} = U^*\{A\}$, $M_{7,A} = \{A \rightarrow A'\}$, and $O_{7,A} = \hat{U}^+$. If $N_{6,B}$ did not delete the last symbol, then the word will be lost since an A is expected now. If the last symbol of the word received is A then the simulation continues, otherwise the word is lost (the previously selected C does not belong to a subword ABC and therefore the rule cannot be applied). In $N_{7,A}$ the symbol A is marked. The word moves to the node $N_{8,A}$ defined by $I_{8,A} = \hat{U}^*\{A'\}$, $M_{8,A} = \{A' \rightarrow \lambda\}$, and $O_{8,A} = U^+$. This node ensures that the A in the end is deleted and not somewhere else. Then a word $u\#v$ has been derived by simulating the rule $ABC \rightarrow \lambda$ where vu is a sentential form of G . The word $u\#v$ is sent to the master node N_0 .

For the rules of the form $S \rightarrow v$, we define an initial node $N_{5,S}$ that prepares the simulation. Let $N_{5,S}$ be defined by the input filter $I_{5,S} = \hat{U}^*\{S'\}$, the set $M_{5,S} = \{S' \rightarrow S_i \mid 1 \leq i \leq p\}$, and the output filter $O_{5,S} = (U \cup \{S_i \mid 1 \leq i \leq p\})^+$. This node chooses a rule P_i that is simulated afterwards. The word obtained is sent to all nodes $N_{6,j}$ with $1 \leq j \leq p$ but it is accepted only by node $N_{6,i}$ which corresponds to the rule selected.

For each rule P_i with $1 \leq i \leq p$, we define the following nodes: the node $N_{6,i}$ is defined by $I_{6,i} = (\{S_i\} \cup U)^+$, $M_{6,i} = \{S_i \rightarrow \lambda\}$, and $O_{6,i} = U^+$. This node deletes S_i (the symbol corresponding to the left hand side of the rule under consideration). If $P_i = S \rightarrow \lambda$ then this word is sent back to the master node, otherwise the rule is $S \rightarrow a_{i,1}a_{i,2} \cdots a_{i,l_i}$ for a natural number $l_i \geq 1$ and symbols $a_{i,j} \in V$ for $1 \leq j \leq l_i$. These symbols will now be appended to the word in a chain of nodes (two nodes for each symbol, one for inserting and one for testing whether the insertion took place in the end of the word). For each number j with $1 \leq j \leq l_i$, we define the nodes $N_{7,i,j'}$ by $I_{7,i,j'} = U^+$, $M_{7,i,j'} = \{\lambda \rightarrow a'_{i,j}\}$, and $O_{7,i,j'} = \hat{U}^+$ as well as $N_{7,i,j}$ by $I_{7,i,j} = \hat{U}^*\{a'_{i,j}\}$, $M_{7,i,j} = \{a'_{i,j} \rightarrow a_{i,j}\}$, and $O_{7,i,j} = U^+$. After the right hand side of the rule P_i is appended, the rule has been simulated, and the word is sent to the master node.

For the case that the marked symbol in $N_{1, \text{sim}}$ is the last symbol and the end marker $\#$, we define the following nodes: $N_{5, \#}$ by $I_{5, \#} = U^* \{\#\}$, $M_{5, \#} = \{\# \rightarrow \lambda\}$, and $O_{5, \#} = U^*$, as well as $N_{6, \#}$ by $I_{6, \#} = T^*$, $M_{6, \#} = \emptyset$, and $O_{6, \#} = T^*$. In $N_{5, \#}$, the end marker is deleted. The word obtained is a sentential form of G and it is sent to node $N_{6, \#}$ which serves as the output node and takes the word only if it is a terminal word.

The NEP $\mathcal{N}$ is defined as

$$\mathcal{N} = (X, N_0, \dots, N_{6, \#}, E, N_{6, \#})$$

with the working alphabet $X = U \cup U' \cup \{S_i \mid 1 \leq i \leq p\}$, the nodes defined above, the set E of edges given in Figure 2, and the output node $N_{6, \#}$. From the explanations given along with the definitions of the nodes and edges, it follows that the network $\mathcal{N}$ generates the same language as the grammar G : $L(\mathcal{N}) = L$. Moreover, all filters are sets of the form AY^* or Y^*A for some alphabet Y and $A \subseteq Y$. Hence, $RE \subseteq \mathcal{E}(BCOMB)$. With Theorem 2.3, $\mathcal{E}(BCOMB) = RE$ follows. $\square$

Completeness can also be achieved if all filters are ordered. Then the completeness follows for networks where all filters are non-counting or all are power-separating.

Theorem 3.21 $\mathcal{E}(ORD) = \mathcal{E}(NC) = \mathcal{E}(PS) = RE$.

Proof. By Lemma 2.2, the inclusions shown in Figure 1, and Theorem 2.3, it is sufficient to prove that $RE \subseteq \mathcal{E}(ORD)$.

Consider the network constructed in the proof of Theorem 3.20. All filters are languages of the form V^* or V^*A or AV^* and can be accepted by deterministic finite automata with one state or the following transition graphs:

These automata have transition functions δ_{COMB} and δ_{BCOMB} given by the following tables:

δ_{COMB}	$a \in A$	$a \in V \setminus A$
z_0	z_1	z_0
z_1	z_1	z_0

δ_{BCOMB}	$a \in A$	$a \in V \setminus A$
z_1	z_1	z_1
z_0	z_1	z_2
z_2	z_2	z_2

It is to be seen that the states in each column for A and $V \setminus A$ preserve the order given in the first column. Hence, all filters are ordered languages. $\square$

It remains open whether also definite or even combinational filters are sufficient for generating all recursively enumerable languages. We conjecture that the use of combinational filters is as powerful as the use of definite filters.

4. Conclusion

The results obtained can be seen in Figure 3.

Figure 3: Hierarchy of the language families generated by NEPs with subregular filters

A solid arrow from X to Y denotes $X \subset Y$ (X is a proper subset of Y), a dashed arrow from X to Y denotes $X \subseteq Y$; if two families are not connected by a directed path then they are not necessarily incomparable. The number at an arrow is the number of the theorem which states the respective inclusion.

We continue the investigations in order to find out whether or not the remaining inclusions are proper and to characterize the families of languages. Especially, regarding the classes $\mathcal{E}(COMB)$ and $\mathcal{E}(DEF)$, we have the conjecture and ideas for a proof that they coincide

References

- [1] A. ALHAZOV, E. CSUHAJ-VARJÚ, C. MARTÍN-VIDE, Y. ROGOZHIN, On the size of computationally complete hybrid networks of evolutionary processors. *Theoretical Computer Science* **410** (2009), 3188–3197.
- [2] A. ALHAZOV, J. DASSOW, C. MARTÍN-VIDE, Y. ROGOZHIN, B. TRUTHE, On networks of evolutionary processors with nodes of two types. *Fundamenta Informaticae* **91** (2009), 1–15.
- [3] J. CASTELLANOS, C. MARTÍN-VIDE, V. MITRANA, J. M. SEMPERE, Solving NP-Complete Problems With Networks of Evolutionary Processors. In: *IWANN '01: Proceedings of the 6th International Work-Conference on Artificial and Natural Neural Networks, LNCS 2084*. Springer-Verlag Berlin, 2001, 621–628.

- [4] J. CASTELLANOS, C. MARTÍN-VIDE, V. MITRANA, J. M. SEMPERE, Networks of Evolutionary Processors. *Acta Informatica* **39** (2003) 6–7, 517–529.
- [5] E. CSUHAJ-VARJÚ, V. MITRANA, Evolutionary systems: A language generating device inspired by evolving communities of cells. *Acta Informatica* **36** (2000) 11, 913–926.
- [6] E. CSUHAJ-VARJÚ, A. SALOMAA, Networks of Parallel Language Processors. In: *New Trends in Formal Languages – Control, Cooperation, and Combinatorics (to Jürgen Dassow on the occasion of his 50th birthday)*. LNCS 1218, Springer-Verlag Berlin, 1997, 299–318.
- [7] L. D. ERRICO, C. JESSHOPE, Towards a new architecture for symbolic processing. In: *AIICSR'94: Proceedings of the sixth international conference on Artificial intelligence and information-control systems of robots*. World Scientific Publishing Co., Inc., River Edge, NJ, USA, 1994, 31–40.
- [8] S. E. FAHLMAN, G. E. HINTON, T. J. SEJNOWSKI, Massively parallel architectures for AI: NETL, Thistle, and Boltzmann machines. In: *Proc. AAAI 1983*. 1983, 109–113.
- [9] V. GEFFERT, Normal forms for phrase-structure grammars. *RAIRO – Theoretical Informatics and Applications* **25** (1991), 473–496.
- [10] I. M. HAVEL, The theory of regular events II. *Kybernetika* **5** (1969) 6, 520–544.
- [11] W. D. HILLIS, *The Connection Machine*. MIT Press, Cambridge, MA, USA, 1986.
- [12] C. MARTÍN-VIDE, V. MITRANA, Networks of evolutionary processors: Results and perspectives. In: *Molecular Computational Models: Unconvent. Approaches*. 2005, 78–114.
- [13] G. PĂUN, Computing with membranes. *J. Comput. Syst. Sci.* **61** (2000) 1, 108–143.
- [14] G. PĂUN, L. SÂNTEAN, Parallel communicating grammar systems: The regular case. *Annals of University of Bucharest, Ser. Matematica-Informatica* **38** (1989), 55–63.
- [15] G. ROZENBERG, A. SALOMAA, *Handbook of Formal Languages*. Springer-Verlag, Berlin, 1997.
- [16] A. SALOMAA, *Formal Languages*. Springer-Verlag, Berlin, 1978.
- [17] D. SANKOFF, G. LEDUC, N. ANTOINE, B. PAQUIN, F. LANG, R. CEDERGREN, Gene order comparisons for phylogenetic inference: Evolution of the mitochondrial genome. *Proceedings of the National Academy of Sciences of the United States of America* **89** (1992) 14, 6575–6579.
- [18] B. WIEDEMANN, *Vergleich der Leistungsfähigkeit endlicher determinierter Automaten*. Diplomarbeit, Universität Rostock, 1978.