# Logik für Informatiker Logic for computer scientists

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# **The Boolean Connectives**

#### Negation — Truth table

# P¬PTRUEFALSEFALSETRUE

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#### The Henkin-Hintikka game



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#### The Henkin-Hintikka game

Is a sentence true in a given world?

- Players: you and the computer (Tarski's world)
- You claim that a sentence is true (or false), Tarski's world will claim the opposite
- In each round, the sentence is reduced to a simpler one
- When an atomic sentence is reached, its truth can be directly inspected in the given world

You have a winning strategy exactly in those cases where your claim is correct.

#### Negation — Game rule

Form	Your commitment	Player to move	Goal
$\neg P$	either	—	Replace $\neg P$ by $P$ and
			switch commitment

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## Conjunction — Truth table

Р	Q	$P \land Q$
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	FALSE

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#### Conjunction — Game rule

Form	Your commitment	Player to move	Goal
	TRUE	Tarski's World	Choose one of $P$ ,
$P \wedge Q$			Q that is false.
	FALSE	you	

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## Disjunction — Truth table

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Р	Q	$P \lor Q$
TRUE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	TRUE	TRUE
FALSE	FALSE	FALSE

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#### Disjunction — Game rule

Form	Your commitment	Player to move	Goal
	TRUE	you	Choose one of $P$ ,
$P \lor Q$			Q that is true.
	FALSE	Tarski's World	

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#### Formalisation

- Sometimes, natural language double negation means logical single negation
- The English expression and sometimes suggests a temporal ordering; the FOL expression ∧ never does.
- The English expressions but, however, yet, nonetheless, and moreover are all stylistic variants of and.
- Natural language disjunction can mean invlusive-or (∨) or exclusive-or: A xor B ⇔ (A ∨ B) ∧ (¬A ∨ ¬B)

## Logical necessity

#### A sentence is

- logically necessary, or logically valid, if it is true in all circumstances (worlds),
- logically possible, or satisfiable, if it is true in some circumstances (worlds),
- logically impossible, or unsatisfiable, if it is true in no circumstances (worlds).

#### Logically possible



#### Logically and physically possible



# Logically impossible $P \land \neg P$ $a \neq a$

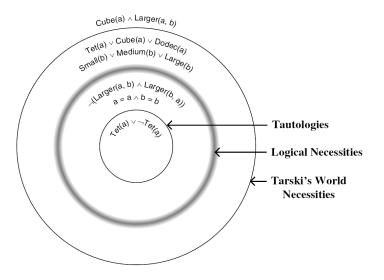
Logically necessary $P \lor \neg P$ a = a

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#### Logic, Boolean logic and Tarski's world

#### A sentence is

- logically necessary, or logically valid, if it is true in all circumstances (worlds),
- TW-necessary, if it is true in all worlds of Tarski's world,
- a tautology, if it is true in all valuations of the atomic sentences with {TRUE, FALSE}.



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